e) $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{8 - x^3}$

$\lim\limits_{x \to 2} \dfrac{x^4 - 16}{8 - x^3} = \lim\limits_{x \to 2} \dfrac{x^4 - 2^4}{2^3 - x^3} = \lim\limits_{x \to 2} \dfrac{(x-2) \cdot (x^3 + x^2 \cdot 2 + x \cdot 2^2 + 2^3)}{(2 - x) \cdot (2^2 + 2 \cdot x + x^2)} =$

$\lim\limits_{x \to 2} \dfrac{\cancel{(x-2)} \cdot (x^3 + 2x^2 + 4x + 8)}{-\cancel{(x-2)} \cdot (4 + 2x + x^2)} = \lim\limits_{x \to 2} \dfrac{(x^3 + 2x^2 + 4x + 8)}{-(4 + 2x + x^2)} =$

$\dfrac{(2^3 + 2 \cdot 2^2 + 4 \cdot 2 + 8}{-(4 + 2 \cdot 2 + 2^2)} = \dfrac{8 + 2 \cdot 4 + 8 + 8}{-(4 + 4 + 4)} =$

$\dfrac{8 + 8 + 8 + 8}{-(12)} = \dfrac{\cancel{32}}{-\cancel{12}} = \dfrac{-8}{3}$

f) $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1} = \lim\limits_{x \to 1} \dfrac{(\sqrt{x} - 1) \cdot (\sqrt{x} + 1)}{(x - 1) \cdot \sqrt{x} + 1} = \lim\limits_{x \to 1} \dfrac{(\sqrt{x})^2 - (1)^2}{(x-1) \cdot (\sqrt{x} + 1)} =$

$\lim\limits_{x \to 1} \dfrac{x - 1}{(x-1) \cdot (\sqrt{x} + 1)} = \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}}{\cancel{(x-1)} \cdot (\sqrt{x} + 1)} = \lim \dfrac{1}{\sqrt{x} + 1} = \dfrac{1}{\sqrt{1} + 1} =$

$\dfrac{1}{1+1} = \dfrac{1}{2}$

g) $$\lim_{x\to -1} \frac{1-x^2}{x-\sqrt{2+x}} = \lim_{x\to -1} \frac{(1^2-x^2)}{(x+\sqrt{2+x})} \cdot \frac{(x-\sqrt{2+x})}{(x-\sqrt{2+x})} =$$

$$\lim_{x\to -1} \frac{(1-x)\cdot(1+x)\cdot(x-\sqrt{2+x})}{x^2-(\sqrt{2+x})^2} =$$

$$\lim_{x\to -1} \frac{(1-x)\cdot(1+x)\cdot(x-\sqrt{2+x})}{x^2-(2+x)} = \lim_{x\to -1} \frac{(1-x)\cdot(1+x)\cdot(x-\sqrt{2+x})}{x^2-2-x}$$

$$\lim_{x\to -1} \frac{(1-x)\cdot(1+x)\cdot(x-\sqrt{2+x})}{(x+1)\cdot(x-2)} =$$

$$\lim_{x\to -1} \frac{(1-x)\cdot\cancel{(1+x)}\cdot(x-\sqrt{2+x})}{\cancel{(1+x)}\cdot(x-2)} =$$

$$\lim_{x\to -1} \frac{(1-x)\cdot(x-\sqrt{2+x})}{(x-2)} =$$

$$\frac{(1-(-1))\cdot(-1-\sqrt{2-1})}{(-1-2)} =$$

$$\frac{(2)\cdot(-1-\sqrt{1})}{-3} = \frac{2\cdot(-2)}{-3} = \frac{4}{3} \#$$

Calculo auxiliar

$x^2-2-x = x^2-x-2$

temos uma equação do 2º grau

$x^2-x-2=0$

$x-x=2$

$x-x+\left(\frac{1}{2}\right)^2 = 2+\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2 = 2+\frac{1}{4}$

$\left(x-\frac{1}{2}\right)^2 = \frac{8+1}{4}$

$\left(x-\frac{1}{2}\right)^2 = \frac{9}{4}$

$x-\frac{1}{2} = \pm\sqrt{\frac{9}{4}}$

$x-\frac{1}{2} = \pm\frac{3}{2}$

$x = \frac{3}{2}+\frac{1}{2}$

$x_1 = 2$

$x_{11} = -\frac{3}{2}+\frac{1}{2}$

$x_{11} = -1$

h) $\lim\limits_{x\to 7} \dfrac{2-\sqrt{x-3}}{x^2-49} = ?$

$$\lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49} = \lim_{x\to 7} \frac{(2-\sqrt{x-3})}{(x^2-7^2)} \cdot \frac{(2+\sqrt{x-3})}{(2+\sqrt{x-3})} =$$

$$\lim_{x\to 7} \frac{2^2-(\sqrt{x-3})^2}{(x-7)\cdot(x+7)\cdot(2+\sqrt{x-3})} = \lim_{x\to 7} \frac{4-(x-3)}{(x-7)\cdot(x+7)(2+\sqrt{x-3})} =$$

$$\lim_{x\to 7} \frac{4-x+3}{(x-7)\cdot(x+7)\cdot(2+\sqrt{x-3})} = \lim_{x\to 7} \frac{(7-x)}{(x-7)\cdot(x+7)\cdot(2+\sqrt{x-3})}$$

$$\lim_{x\to 7} \frac{\cancel{-(x-7)}}{\cancel{(x-7)}\cdot(x+7)\cdot(2+\sqrt{x-3})} = \lim_{x\to 7} \frac{-1}{(x+7)\cdot(2+\sqrt{x-3})} =$$

$$\frac{-1}{(7+7)\cdot(2+\sqrt{7-3})} = \frac{-1}{14\cdot(2+\sqrt{4})} = \frac{-1}{14\cdot(2+2)} =$$

$$\frac{-1}{14\cdot(4)} = -\frac{1}{56}$$

i) $$\lim_{x \to 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} =$$

$$\lim_{x \to 4} \frac{(3 - \sqrt{5+x})}{(1 - \sqrt{5-x})} \cdot \frac{(3 + \sqrt{5+x})}{(1 + \sqrt{5-x})} \cdot \frac{(1 + \sqrt{5-x})}{(3 + \sqrt{5+x})} =$$

$$\lim_{x \to 4} \frac{(3^2 - (\sqrt{5+x})^2) \cdot (1 + \sqrt{5-x})}{(1^2 - (\sqrt{5-x})^2) \cdot (3 + \sqrt{5+x})} =$$

$$\lim_{x \to 4} \frac{9 - (5+x) \cdot (1 + \sqrt{5-x})}{(1 - (5-x)) \cdot (3 + \sqrt{5+x})} =$$

$$\lim_{x \to 4} \frac{(9 - 5 - x) \cdot (1 + \sqrt{5-x})}{(1 - 5 + x) \cdot (3 + \sqrt{5+x})} =$$

$$\lim_{x \to 4} \frac{(4 - x) \cdot (1 + \sqrt{5-x})}{(-4 + x) \cdot (3 + \sqrt{5+x})} =$$

$$\lim_{x \to 4} \frac{\cancel{(4 - x)} \cdot (1 + \sqrt{5-x})}{-\cancel{(4 - x)} \cdot (3 + \sqrt{5+x})} = \lim_{x \to 4} \frac{(1 + \sqrt{5-x})}{-(3 + \sqrt{5+x})} = \frac{(1 + \sqrt{5-4})}{-(3 + \sqrt{5+4})} =$$

$$\frac{(1 + \sqrt{1})}{-(3 + \sqrt{9})} = \frac{(1+1)}{-(3+3)} = \frac{2}{-6} = -\frac{2}{6}$$